TEST - 01

MODERN ALGEBRA

16 April, 2022   Time - 2 hours

Topic - Set, Relation, function, Number Theory, Group.

1. Read all the questions carefully atleast two times.

2. In Part-A each correct answer carry 02 marks and incorrect answer has negative marking -01.

3. In Part-B each correct answer carry 02 marks and no negative marking for incorrect answer.

Part-A(Single Correct Questions)

1. The relation R on the set $\mathbb{R}$ of all real numbers, defined as $R = \{(a, b)|a \leq b^2\}$ is
   a. reflexive   b. symmetric   c. transitive   d. none

2. $f\colon A \to B$, $g\colon B \to C$ be two functions, if $g \circ f\colon A \to C$ is one-one. Then
   a. f is one-one   b. g is one-one   c. both one-one   d. none

3. How many divisors of 10
   a. 2   b. 4   c. 8   d. 1

4. gcd(-8, 0) is
   a. 1   b. -8   c. 8   d. 0

5. Which of the following is relatively prime to each other
   a. (2, 8)   b. (3, 9)   c. (4, 13)   d. (4, 16)

6. $f\colon \mathbb{R} \to \mathbb{R}^+$, $f(x) = 1 + x^2$ is
   a. one-one   b. onto   c. bijective   d. none

7. Number of 3×3 non-singular matrices over the field $\mathbb{F}_2$
   a. 168   b. 384   c. 2^3 d. 3^2

8. $f = (123)(1456)$ then f^{58} is equal to
   a. (132654)   b. (123)(456)   c. (125)(364)   d. (152)(346)

9. $\beta^9 = (2143567) \in S_7$ then β is
   a. (2143567)   b. (2547136)   c. (2457136)   d. (2457163)

10. Number of elements of order 2 in S_6
    a. 6   b. 7   c. 8   d. 5

11. If $O(a) = n$, $a \in G$ such that $a^k = e$. Then
    a. $k|n$ b. $n|k$ c. $n = k$ d. $n \neq k$

12. Let $G = \{\alpha, 1, 3, 9, 19, 27\}$ is cyclic group of order 6 with respect to multiplication modulo 56. Then
    a. $\alpha = 5$ b. $\alpha = 15$ c. $\alpha = 25$ d. $\alpha = 35$

13. Let G be a cyclic group and G has exactly one generator. Then G has elements
    a. finite   b. infinite   c. at most 2   d. at least 2

14. Let $O(G) = 122$, then number of non-isomorphic group
    a. 2   b. 1   c. 61   d. 4

15. Let $O(G) = 77$. Then G is
    a. always abelian   b. abelian but not cyclic   c. non abelian
    d. none

16. If $O(G) = 81$. Then find number of non-isomorphic abelian group of order 81
    a. 3   b. 4   c. 5   d. 9

17. If $O(G) = 16$ and G has exactly 3 elements of order 2, then number of elements of order 4 in G
    a. 4   b. 6   c. 8   d. 12

Part-B (Numerical Answer Type Questions)

18. Let $f\colon \mathbb{R} \to \mathbb{R}$, $g\colon \mathbb{R} \to \mathbb{R}$ such that $f(x) = x^2 + 2x - 3$, $g(x) = 3x - 4$. Then find $f \circ g(x)$ and $g \circ f(x)$.

19. Let $f\colon A \to B$ with $|A| = 3$ and $|B| = 2$. Then find total number of one-one functions and total number of onto functions.

20. Let $f\colon A \to B$ with $|A| = 2$ and $|B| = 3$. Then find total number of one-one functions and total number of onto functions.

21. Let $f\colon A \to B$ with $|A| = 3$ and $|B| = 3$. Then find total number of bijective functions.

22. gcd(1034, 132) is ?

23. $\sigma(1020)$ is ?

24. The highest order of any element $f \in S_9$ is ?

25. Number of elements of order 2 in A_6.

26. Number of elements of order 2 in $S_4 \times A_4$.

27. What is the order of $(R_{90}, 3, (23), 4)$ in $D_4 \times Z_5 \times S_5 \times Z_{16}$?

28. $\mathrm{lcm}(\sigma(10), \tau(20)) \cdot \gcd(\omega(10), \phi(20)) = ?$

29. If $O(G) = 8$ and G has 7 elements of order 2. Then G is isomorphic to which group ?

30. Let $G = D_4$, then $D' \cdot R_{90} \cdot H \cdot R_{270} \cdot V \cdot R_{180} \cdot D$ is ?

Best wishes from Vivek Sahu.