TEST - 02

MODERN ALGEBRA

02 May, 2022 Time - 2 hours
maximum marks - 60

Topic - Number Theory, Group, Subgroup Normal Subgroup, Factor Group.

INSTRUCTIONS:

1. Read all the questions carefully atleast two times.
2. In Part-A (single correct questions) each correct answer carry 02 marks and incorrect answer has negative marking -01.
3. In Part-B (multiple correct questions) each correct answer carry 03 marks and no negative marking for incorrect answer.

Part-A(Single Correct Questions)

1. The set $\{5, 15, 25, 35\}$ is group with respect to multiplication modulo 40. The identity element of this group is
   a. 5 b. 15 c. 25 d. 35
2. Which of the following is false
   a. any abelian group of order 27 is cyclic
   b. any abelian group of order 14 is cyclic
   c. any abelian group of order 21 is cyclic
   d. any abelian group of order 30 is cyclic
3. Let G be a cyclic group of order n and m is a divisor of n. Then
   a. G contains a unique subgroup of order of m
   b. G may or may not contains subgroup of order m
   c. G must contain a subgroup of order m, but not unique
   d. G can not contains any subgroup of order m
4. Let G be group of order 77. Then center of G is isomorphic to
   a. $\mathbb{Z}_1$ b.$\mathbb{Z}_7$ c. $\mathbb{Z}_{11}$ d. $\mathbb{Z}_{77}$
5. Let G be a non abelian group. Then order of G can be
   a. 25 b. 55 c. 125 d. 35

6. Let G be a group with the generator a and b given by $G = \{a, b | a^4 = b^2 = e, ba = a^{-1}b\}$. if $Z(G)$ denote the center of G, then $\frac{G}{Z(G)}$ is isomorphic to
   a. trivial group   b. $\mathbb{Z}_2$ c. $\mathbb{Z}_2 \times \mathbb{Z}_2$ d. $\mathbb{Z}_4$

7. If $Z(G)$ denote the center of G. Then $\frac{G}{Z(G)}$ can not be
   a. 4   b. 6   c. 15   d. 25

8. Upto isomorphic abelian group of order 109
   a. 12   b. 9   c. 6   d. 5

9. Number of proper subgroups of the group $\mathbb{Z} \times \mathbb{Z}$ is
   a. 1   b. 2   c. 3   d. infinitely many

10. Number of elements of order 10 in $\frac{Q}{Z}$
    a. 10   b. 5   c. 4   d. 2

11. $a = 10, b = 25$. Then $gcd[\phi(a), \tau(b)] \cdot lcm[\sigma(a), \omega(b)]$
    a. 1   b. 12   c. 12×18 d. 18

Part-B (Multiple Correct Questions)

12. Which of the following is/are true
    a. $\mathbb{Z}_2 \times \mathbb{Z}_3 \cong \mathbb{Z}_6$ b. $\mathbb{Z}_4 \times \mathbb{Z}_6 \cong \mathbb{Z}_{24}$
    c. $\mathbb{Z}_3 \times \mathbb{Z}_3 \cong \mathbb{Z}_9$ d. $\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5 \cong \mathbb{Z}_{30}$

13. Which of the following is/are abelian group
    a. $(\mathbb{Z}, +)$ b. $(\mathbb{R}, +)$
    c. $(M_(m \times n)(\mathbb{R}), +)$ d. $(M_(m \times n)(\mathbb{R}), \cdot)$

14. Which of the following is/are not simple group
    a. $\mathbb{Z}_5$ b. $\mathbb{Z}_6$ c. $\mathbb{Z}_2$ d. $\mathbb{Z}_8$

15. Which of the following is/are true
    a. every group of order 36 is abelian
    b. a group in which every element of order at most 2 is abelian
    c. every group of order 36 is non abelian
    d. none of the above

16. Let G be a non abelian group and $a, b \in G$ such that order of a is 4 and order of b is 3. Then order of $a \cdot b$ is
    a. 6   b. 12   c. $12k, k \geq 2$ d. need not be finite

17. Which of the following have same order
    a. $\frac{\mathbb{Z}_3 \times \mathbb{Z}_5}{<(1,2)>}$ b. $\frac{\mathbb{Z}_4 \times \mathbb{Z}_6}{<(2,3)>}$ c. $\frac{\mathbb{Z}_5 \times \mathbb{Z}_10}{<(1,5)>}$ d. $\frac{\mathbb{Z}_4 \times \mathbb{Z}_4}{<(1,3)>}$

18. Which of the following is/are normal subgroup
    a. kernal   b. center   c. index two   d. all of the above

19. Let $G = \{I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, C = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\}$
be a multiplicative group of matrices. Then
a. $O(A) = O(B) = O(c) = 2$
b. G is cyclic group
c. G is abelian group
d. G is non abelian group

20. Which of the following is/are not true
a. A_n is normal in $S_n, n \geq 4$
b. K_4 is normal in $A_n, n \geq 4$
c. K_4 is normal in $S_n, n \geq 4$
d. $\{e\}$ is normal in $A_n, n \geq 4$

21. Which of the following is/are true
a. $\mathbb{Z}_n$ is cyclic iff n is prime
b. every proper subgroup og $\mathbb{Z}_n$ is cyclic
c. every proper subgroup of S_4 is cyclic
d. if every proper subgroup of G is cyclic then G is cyclic

22. Let H be the quotient group $\frac{Q}{Z}$, consider the statement
a. every cyclic subgroup of H is finite
b. every finite cyclic group is isomorphic to a subgroup of H
c. both
d. none

23. Which of the following is/are of order 5 in $\frac{Q}{Z}$
a. 1/5 b. 5/4 c. 5/1 d. 3/15 e. 4/5

Bonus Question(2 marks)
how many group homomorphism $f: S_4 \to Z_4$?

Best wishes from V!vek Sahu.