TEST-03

MODERN ALGEBRA

Date - Tuesday, 17may, 2022 Max. Marks - 60

TOPIC - Homomorphism, Class equation, Sylow's theorem.

INSTRUCTIONS:

1. Read all the questions carefully atleast two times.
2. In Part-A (single correct questions) each correct answer carry 02 marks and incorrect answer has negative marking -01.
3. In Part-B (multiple correct questions) each correct answer carry 03 marks and no negative marking for incorrect answer.

PART-A

1. How many onto homomorphism $f : Z_{16} \to Z_4 \times Z_2$
   (a) 8 (b) 4 (c) 2 (d) 1
2. $f : G_1 \to G_2$ be a group homomorphism and $O(G_1) = 10, O(G_2) = 25$.Then possible order of kernal of f is
   (a) 1 (b) 2 (c) 3 (d) 4
3. The order of $Inn(Z_{10})$ is
   (a) 10 (b) 2 (c) 1 (d) 0
4. If $f : D_4 \to D_4$, then how many inner-automorphism
   (a) 8 (b) 4 (c) 2 (d) 1
5. Let H and K are two subgroups of order 3 and 5 respectively. If order of G is 15 then
   (a) $H \cap K = \{e\}$ (b) $H \cap K = K$
   (c) $H \cap K = H$ (d) $H \cap K \neq \{e\}$
6. Which of the following can be class equation of group of order 10.
   (a) $1 + 1 + 1 + 2 + 5$ (b) $1 + 2 + 2 + 5$
   (c) $2 + 2 + 2 + 2 + 2$ (d) $1 + 1 + 2 + 2 + 2 + 2$

7. Consider $T : Z_{12} \to Z_{12}$ the number of inner-automorphism is equal to
   (a) 1 (b) 2 (c) 4 (d) 12
8. The number of 5-sylow subgroups in S_6 is
   (a) 16 (b) 6 (c) 36 (d) 1
9. The number of 5-sylow subgroups in the group of order 45 is
   (a) 1 (b) 2 (c) 3 (d) 4
10. Let G be a group of order 60, then
    (a) G is abelian
    (b) G has subgroup of order 30
    (c) G has subgroup of order 2, 3, 5
    (d) G has subgroup of order 6, 10, 15
11. How many 17-sylow subgroups are in Group of order 595
    (a) 1 (b) 2 (c) 17 (d) 35
12. Which of the following number can be order of permutation σ of 11 symbols such that σ does not fix any symbol
    (a) 15 (b) 18 (c) 28 (d) 30
13. Let H_1 and H_2 be two distinct subgroups of a finite group G, each of order 2. Let H be the smallest subgroup containing H_1 and H_2. Then the order of H is
    (a) always 2 (b) always 4 (c) always 8 (d) None
14. Total number of conjugate class in A_5 is
    (a) 4 (b) 5 (c) 8 (d) none of these
15. Let G be a simple group of order 168. Then the number of subgroups of G of order 7
    (a) 1 (b) 7 (c) 8 (d) 28
16. Let G be a group of order 200, then the number of subgroups of G of order 25 is
    (a) 1 (b) 4 (c) 5 (d) 10

17. Let G be group or order 1225, then G is
(a) abelian but not cyclic (b) cyclic
(c) simple group (d) none of the above

18. Number of Automorphism from D_5 is
(a) 25 (b) 5 (c) 20 (d) 5.5!

19. In the permutation group $S_n (n > 4)$, if H is the smallest subgroup containing all 3-cycles then, which one of the following is true
(a) Order of H is 2 (b) Index of H in S_n is 2
(c) H is abelian (d) $H = S_n$

20. Let S_9 be the group of all permutations of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Then the total number of elements in S_9 that commute with $\tau = (123)(4567)$ in S_9 equals
(a) $9 \times 8 \times 7 \times 6 \times 5$ (b) 9! (c) 24 (d) none

PART-B

1. Consider the symmetric group S_{20} and it's subgroup A_{20}. Let H be a 7-ssg of A_{20}. Pick the correct statement from below.
(a) $O(H) = 7^2$
(b) H must be cyclic
(c) H is normal subgroup of A_{20}
(d) Any 7-ssg of S_{20} is subgroup of A_{20}

2. Let a_n denote the number of those permutation σ on $\{1, 2, 3, ...\}$ such that σ is a product of disjoint cycle then.
(a) $a_5 = 50$ (b) $a_4 = 14$ (c) $a_5 = 40$ (d) $a_4 = 11$

3. Let G be a group if order 231. Then the number of elements of order 11 is
(a) 1 (b) 10 (c) 11 (d) 21

4. For any prime p, consider the group $G = GL_2[Z_p]$. Then which of the followings are true

(a) G has an element of order p
(b) G has exactly one element of order p
(c) G has no p-ssg
(d) b and c are correct

5. Let S_5 be the symmetric group. Then which of the following statements is false
(a) S_5 contains a cyclic subgroup of order 6
(b) S_5 contains a non-Abelian subgroup o order 8
(c) S_5 does not contain a subgroup isomorphic to Z_2 x Z_2
(d) S_5 does not contain a subgroup of order 7

6. G be a simple group of order n, then value of 'n' can never be
(a) 27 (b) 60 (c) 125 (d) 360

Bonus Question: True or False
Q. S_n is isomorphic to a subgroup of A_{n+2}.

....Best wishes from V!vek Sahu....